ESTIMATION OF STOCHASTIC PARAMETERS OF INERTIAL SENSORS USING KALMAN FILTER

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Summary. Accelerometers and gyroscopes as basic information sources of sepcific forces and angular rates, which are input information for inertial navigation systems, produce these information with errors. The area of deterministic errors, which can be supressed by calibration, does not represent the solution of inaccuracy influence of inertial sensors output on quality and accuracy of navigation information. The stochastic errors have, in terms of time evolution, enormous influence on accuracy of inertial navigation systems. The content of this contribution is estimation of bias variation of inertial sensor, which is categorized as stochastic error. The estimation using Kalman filter is widely used method for supressing of bias variation by its estimation and correction in sensors output.

Keywords: inertial sensor; bias drift; Kalman filter

1. INTRODUCTION

Despite improvement of pruduction techniques and application of advanced physical princpiples during design of inertial sensors, their outputs are influenced by various errors. The basic error division of inertial sensors is according to [1] and [4] on deterministic and stochastic. The deterministic errors – bias, scale factor and misalignment – are errors, which can be supressed or nearly completely removed by calibration. The deterministic errors are removed and supressed by mathematical and measurement techniques, while stochastic errors are random. It is necessary to choose other way of determination of parameters than in deterministic way.

The basic methods of stochastic parameters determination are:
- Allan variance
- Fourier analysis
- Power spectral density (PSD)

The aggregate property of these methods is analysis of stochastic error part, which can be determined using predefined mathematical steps. All of these methods are leading to determination of stochastic signal structure, either by visualization of all harmonic frequencies or by visualization of noise types in way of curves slopes. These mathematic tools enables determination of stochastic mechanisms coefficients, but they are only intermediate level of algorithms for supressing of stochastic processes in real time during solution of inertial or integrated navigation process. When we focus on bias variation of inertial sensor, according to random processes is used Gauss – Markov process of the first order, and for this process is necessary knowledge of time constant $T = 1/\beta$, standard deviance of the driving noise σ_{GM}^2, sampling Δt and driving noise w_k itself, Discrete model is as follows.

$$x_{k+1} = e^{-\beta\Delta t} x_k + \sqrt{\sigma_{GM}^2 \left(1 - e^{-2\beta\Delta t}\right)} w_k \tag{1}$$

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2. INERTIAL SENSOR ERROR MODEL

According to [2] there is a lot of ways, how to define sensor output mathematically. In this article, following model is applied for sensor output.

$$out_{\omega} = \text{info} + b_{\omega} + b_{\omega VAR} + \varepsilon_{\omega} \tag{2}$$

In equation (2) is angular rate sensor output out_{ω}, real uninfluenced information info, bias of sensor b_{ω}, bias variation $b_{\omega VAR}$ and wide band noise ε_{ω}. The bias variation is based in this case on equation (1). Consider angular rate sensor, which output consists also stochastic part with following parameters setup.

Table 1 Angular rate sensor (ARS) parameters of stochastic part

Parameter		Value	Unit
Angular rate sensor time constant (x)	T_X	10 000	sec
Angular rate sensor time constant (y)	T_Y	10 000	sec
Angular rate sensor time constant (z)	T_Z	10 000	sec
Variance of Gauss – markov process driving noise (x)	$\sigma^2_{GM(X)}$	0.01	rad/sec
Variance of Gauss – markov process driving noise (y)	$\sigma^2_{GM(Y)}$	0.015	rad/sec
Variance of Gauss – markov process driving noise (z)	$\sigma^2_{GM(Z)}$	0.02	rad/sec
Variance of wide – band noise (x)	$\sigma^2_{\varepsilon(X)}$	0.01	rad/sec
Variance of wide – band noise (y)	$\sigma^2_{\varepsilon(Y)}$	0.01	rad/sec
Variance of wide – band noise (z)	$\sigma^2_{\varepsilon(Z)}$	0.01	rad/sec
Sampling	Δt	0.01	sec

In following simulation we consider stationary sensor output, which in this case is ideally calibrated. This level of stochastic noise part is familiar for MEMS based angular rate sensors, which also suffer not only by these two errors – wide band noise and bias variation. Following simulated outputs are the result of Table 1 setup.

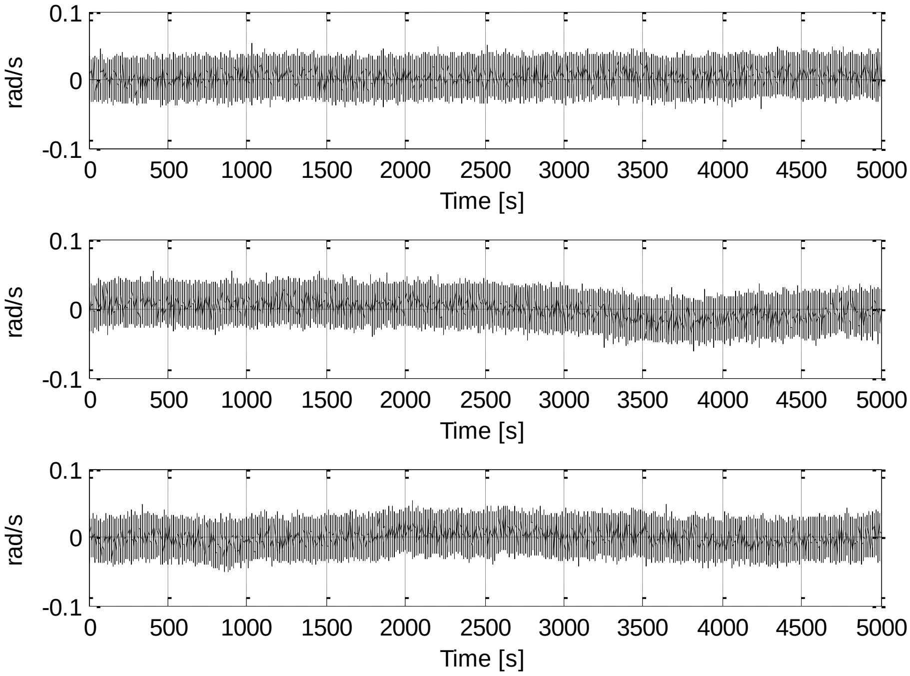

Figure 1 Stationar angular rate sensor output based on Table 1

For better graphical explanation is used Allan variance, which enables determination of stochastic mechanisms based on slopes of parts of resulting curve. This standardized tool for sensor description is better explained in [5].

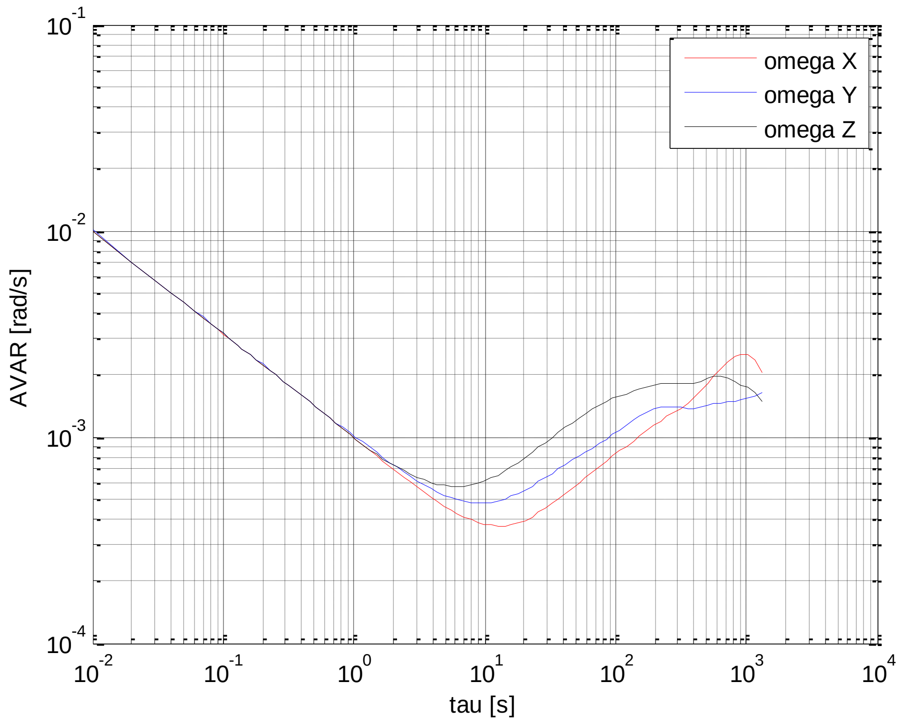

Figure 2 Allan variance of angular rate sensor output

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3. ESTIMATION OF BIAS VARIATION USING KALMAN FILTER

There is a lot of ways how to define Kalman filter algorithm [3]. According to short description is mathematical algorithm of Kalman filter a process of state estimation in linear dynamic systems, which are corrupted by white Gaussian noise. The measurements of these processes are linear functions of thsese states and as well are corrupted by white Gaussian noise. The nonlinear cases are solved using linearized (LKF) or extended (EKF) filter. The algorithm is divided on prediction and correction part. For state estimation is also required state and measurement description of dynamic system. The state description in this case is based on three Gauss – markov procesess also with defined parameters in continuous time.

$$\begin{bmatrix} \dot{b}_{\omega VAR(X)} \\ \dot{b}_{\omega VAR(Y)} \\ \dot{b}_{\omega VAR(Z)} \end{bmatrix} = \begin{bmatrix} -\beta_1 & 0 & 0 \\ 0 & -\beta_2 & 0 \\ 0 & 0 & -\beta_3 \end{bmatrix} \begin{bmatrix} b_{\omega VAR(X)} \\ b_{\omega VAR(Y)} \\ b_{\omega VAR(Z)} \end{bmatrix} + \begin{bmatrix} \sqrt{2\beta_1\sigma^2_{GM(X)}} & 0 & 0 \\ 0 & \sqrt{2\beta_1\sigma^2_{GM(Y)}} & 0 \\ 0 & 0 & \sqrt{2\beta_1\sigma^2_{GM(Z)}} \end{bmatrix} \begin{bmatrix} w(t)_X \\ w(t)_Y \\ w(t)_Z \end{bmatrix} \tag{3}$$

Description of measurements:

$$\begin{bmatrix} y(t)_X \\ y(t)_Y \\ y(t)_Z \end{bmatrix} = H \begin{bmatrix} b_{\omega VAR(X)} \\ b_{\omega VAR(Y)} \\ b_{\omega VAR(Z)} \end{bmatrix} + \begin{bmatrix} v(t)_X \\ v(t)_Y \\ v(t)_Z \end{bmatrix} \tag{4}$$

Kalman filter based on (3) and (4) leads to estimation of bias variation and subsequent correction.

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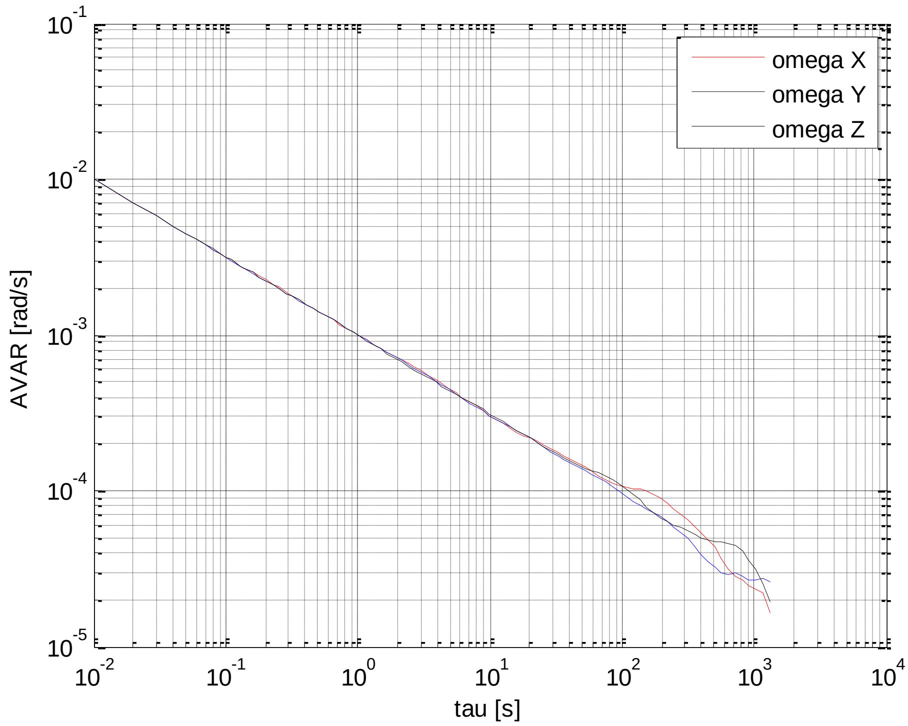

Figure 3 Allan variance after correction by estimated bias variation

The procedure above also with state and measurement description are widely used forms of bias variation estimation and subsequent correction, leading to reduction of navigation errors according to uncertanities caused by bias variation of sensors. In other case would significantly affect not only accuracy of integrated or inertial navigation solution, but also convergency and stability of applied Kalman filter. As an example of direct application is below shown error state vector of extended Kalman filter (EKF), whoch consists of error in position $\delta\dot{p}^{3x1}$, velocity $\delta\dot{v}^{3x1}$ and attitude $\delta\dot{\eta}^{3x1}$ but also has additional states $\dot{b}_{\omega VAR}^{3x1}$ and $\dot{b}_{fVAR}^{3x1}$ for estimation of accelerometers and angular rate sensors bias variation. Closer explanation in [3][6].

$$\dot{x} = \begin{bmatrix} \delta\dot{p}^{3x1} & \delta\dot{v}^{3x1} & \delta\dot{\eta}^{3x1} & \dot{b}_{\omega VAR}^{3x1} & \dot{b}_{fVAR}^{3x1} \end{bmatrix}^T \tag{5}$$

4. CONCLUSION

The content of this contribution is application of Kalman filter in field of estimation and correction of angular rate sensor bias variation. This stochastic error is modelled by Gauss – Markov process of the first order, described by (1). The tool for demonstration of this stochastic error supression is Allan variance. As is visible on curves comparison in Fig. 2 and Fig. 3, after correction, which is just simple subtraction of estimated bias variation form sensor output in Fig. 1, the result is change in shape of curves. Before correction is clearly visible minimum in curves of Allan variance. This the demonstration of bias instability – variation of bias during long time period. The estimation of bias variation by Kalman filter and its correction is noticeable in Fig. 3. In this picture sensor appears without bias variation and slope of Allan variance describes wide band noise. This ideal example clearly demonstrates high level of bias variation supression abilities using estimation algorithm. Real output of inertial sensor is also corrupted by more than two stochastic errors. Long term stationary measurements and determination of all noise mechanisms would lead to high accuracy of bias variation supression.

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