FLOW MODELING BY HIGH-PERFORMANCE COMPUTING

Peter SZABÓ*, Peter GAŠPAROVIČ, Miroslava FERENCOVÁ
Technical University of Košice, Faculty of Aeronautics, Rampová 7, 041 21 Košice, Slovak Republic
**Corresponding author.* E-mail: peter.szabo@tuke.sk

Summary. Flow modeling is used to study and analyse the flow around the examined items. This complex process is also based on the use meshes - special graphs that were created (vertices and edges) using mathematical functions. Upon decomposition the mesh for parallel calculations, computational part of the process is carried out using high-performance computing.

The work describes the process flow modeling and a way how to apply the process for using high performance computing system OpenFOAM in conditions of the Technical University in Košice.

Keywords: flow; numerical simulation; CFD; high-performance computing; OpenFOAM; SageMath;

1. FLOW MODELING

On computers process flow modeling has the following steps:

a) input data definition
b) defining the geometry
c) defining the mesh
d) decomposition of geometry
e) aerodynamic computations in mesh points
f) evaluation and analysis results of computations

The steps a), b), c), d) called together *pre-processing*. During the pre-processing, we need to define a mathematical model of the examined object (geometry) and its surroundings (mesh).
Step e) (*the actual computations*) is used to perform and record the results of aerodynamic computations in mesh points in the given time $t_0, t_1, \dots, t_n$. This step is very difficult to computation, it is therefore used in high-performance computing and parallel programming.
In step f) (*post-processing*) are processed outputs from point e), these outputs can create animation by time and by examined individual variables.

In the article we will mainly deal with pre-processing and with the question of how to design a simple two-dimensional geometry of the wing profile of the aircraft. Note, this two-dimensional geometry can be extended to 3D.

2. INPUT DATA

2D wing profile of aircraft P will be specified as a sequence of points in the plane $[x_i, y_i]$, $i = 0, \dots, m$. These points define the polygon. The more points (greater m) will be determined more accurate profile. In addition to these points, it is necessary to enter the area in which will be examined the aerodynamic events and changes. For simplicity, it will be a rectangle, denoted by O, which is defined by two points $[U_1, U_2]$ (upper left corner) and $[D_1, D_2]$ (lower right corner). Points for a wing profile are restrictions: $U_1 \le x_i \le D_1$ and $D_2 \le y_i \le U_2$, for $i = 0, \dots, m$.
Note, in this way can be given any profile not only a wing profile.

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice

3. DEFINITION OF THE PROBLEM

Our task is to define the mesh (grid) of points in rectangle *O* around the wing profile *P* (based on the input data) :

$$[x^k_i, y^k_i]$$

where $i = 0, 1,\dots,m$ and $k = 1, 2,\dots,n$ define the different levels (layers) of the mesh.

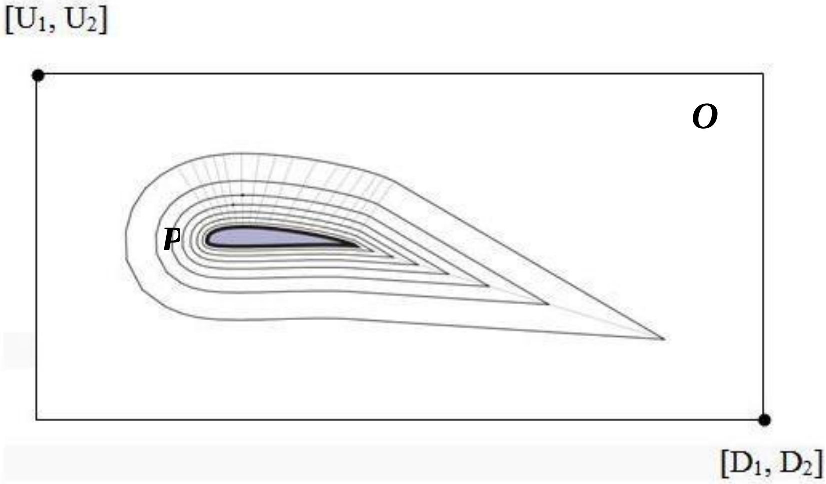

Figure 1 Wing profile and the mesh

4. A MESH ALGORITHM

This section describes an algorithm that assigns a grid to profile *P* in *O*. The steps of our algorithm:

0) i=0
a) Define a further trio of profile points $[x_{i-1}, y_{i-1}]$, $[x_i, y_i]$, $[x_{i+1}, y_{i+1}]$,
when point $[x_i, y_i]$ is not so singular
- b_1) assigning a quadratic function $f(x) = ax^2 + bx + c$.
- c_1) Compute the tangent
- d_1) Compute the perpendicular (normal)

when point $[x_i, y_i]$ is so singular
- b_2) Rotates the coordinate system by 90 degree.
- c_2) assigning a quadratic function
- d_2) Compute the tangent

e) define the layers of grid for k = $1, 2,\dots,n$.
f) if $i < m$ then $i{+}{+}$ and goto a)

Notes:
1) The algorithm works correctly when the coefficient of a quadratic function $a \neq 0$. When the coefficient $a = 0$ the point $[x_i, y_i]$ may be omitted since this point located on a line defined by the points $[x_{i-1}, y_{i-1}]$ and $[x_{i+1}, y_{i+1}]$.
2) Let us put $x_{m+1}=x_0$, $y_{m+1} = y_0$ and $x_{-1} = x_m$, $y_{-1} = y_m$ thus is guaranteed initialization and termination of algorithm.
3) We assume, without loss of generality, that $[x_i \neq x_j]$ for $i \neq j$.

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice

4.1. Singular points

A point of profile $[x_i, y_i]$ is *singular* if and only if the following applies:

$$x_{i-1} < x_i > x_{i+1} \text{ or}$$
$$x_{i-1} > x_i < x_{i+1}.$$

This definition implies that the point of profile $[x_i, y_i]$ is *not singular* if and only if:

$$x_{i-1} < x_i < x_{i+1} \text{ or}$$
$$x_{i-1} > x_i > x_{i+1}.$$

4.2. A quadratic function

If a point of profile $[x_i, y_i]$ is *not singular* then we consider points $[x_{i-1}, y_{i-1}]$, $[x_i, y_i]$, $[x_{i+1}, y_{i+1}]$, for $i = 0, 1, \ldots, m$. The three points define a quadratic function $f(x) = ax^2 + bx + c$. We are looking for the value of a, b, c and the equation $r_i(x) = p_i x + q_i$ the perpendicular (normal line) of function $f(x)$ in point x_i. By using the function $r_i(x)$ define the layers $k = 1, \ldots, n$ of our mesh.

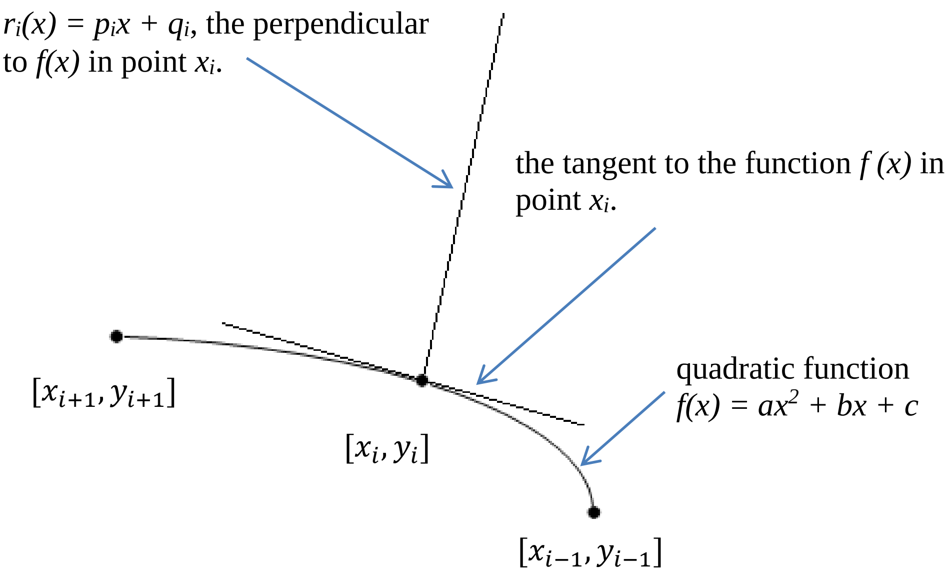

Figure 2 A quadratic function and the mesh construction

Points $[x_{i-1}, y_{i-1}]$, $[x_i, y_i]$, $[x_{i+1}, y_{i+1}]$ lie on the function $f(x)$ therefore applies following system of equations:

$$\begin{aligned} y_{i-1} &= a * x_{i-1}^2 + b * x_{i-1} + c \\ y_i &= a * x_i^2 + b * x_i + c \\ y_{i+1} &= a * x_{i+1}^2 + b * x_{i+1} + c \end{aligned}$$

The coefficients a, b, c of function $f(x)$ are the solution of this system. System can be entered in the matrix form and to solve by Cramer rules.

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice

$$A = \begin{pmatrix} x_{i-1}^2 & x_{i-1} & 1 \\ x_i^2 & x_i & 1 \\ x_{i+1}^2 & x_{i+1} & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$Y = \begin{pmatrix} y_{i-1} \\ y_{i1} \\ y_{i+1} \end{pmatrix}$$

The solution of this system are the following values:

$$a = \frac{(x_{i-1})*(y_{i+1}-y_i)+(x_i)*(y_{i-1}-y_{i+1})+(x_{i+1})*(y_i-y_{i-1})}{(x_{i-1}^2)*(x_i+x_{i+1})+(x_i^2)*(x_{i+1}-x_{i-1})+(x_{i+1}^2)*(x_{i-1}-x_i)}$$

$$b = \frac{(y_{i-1})*(x_{i+1}^2-x_i^2)+(y_{i+1})*(x_i^2-x_{i-1}^2)+(y_i)*(x_{i-1}^2-x_{i+1}^2)}{x_{i-1}^2*(x_i+x_{i+1})+x_i^2*(x_{i+1}-x_{i-1})+x_{i+1}^2*(x_{i-1}-x_i)}$$

$$c = \frac{(x_{i-1}^2)*(x_i*y_{i+1}-x_{i+1}*y_i)+(x_i^2)*(x_{i+1}*y_{i-1}-x_{i-1}*y_{i+1})+(x_{i+1}^2)*(x_{i-1}*y_i-x_i*y_{i-1})}{x_{i-1}^2*(x_i+x_{i+1})+x_i^2*(x_{i+1}-x_{i-1})+x_{i+1}^2*(x_{i-1}-x_i)}$$

4.3. Tangent and perpendicular

Equation of the *tangent* to the graph of function *f(x)* at point $[x_i, y_i]$:

$$y = f'(x_i)(x - x_i) + f(x_i),$$

where $f'(x_i)$ is the derivation of *f(x)* at point x_i.
We obtain after substitution:

$$y = (2ax_i + b) * (x - x_i) + (ax_i + bx_i + c)$$
$$y = 2ax_i x - 2ax_i^2 + bx - bx_i + ax_i^2 + bx_i + c$$
$$y = -1ax_i^2 + 2ax_i x + bx + c$$
$$\boldsymbol{y = (2ax_i + b)x - ax_i + c} \qquad (1)$$

Equation of the *perpendicular* to the graph of function *f(x)* at point $[x_i, y_i]$:

$$y = -\frac{1}{f'(x_i)}(x - x_i) + f(x_i)$$

After similar modifications as in case tangent we get the equation of perpendicular:

$$y = -\frac{1}{2ax_i + b}(x - x_i) + ax_i^2 + bx_i + c$$
$$y = \frac{x_i - x}{2ax_i + b} + ax_i^2 + bx_i + c$$
$$\boldsymbol{y = -\frac{1}{2ax_i + b}x + \frac{x_i}{2ax_i + b} + ax_i^2 + bx_i + c} \qquad (2)$$

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice

These considerations have applied to the not singular points. Now we consider the next case, if points $[x_{i-1}, y_{i-1}]$, $[x_i, y_i]$, $[x_{i+1}, y_{i+1}]$ are singular.

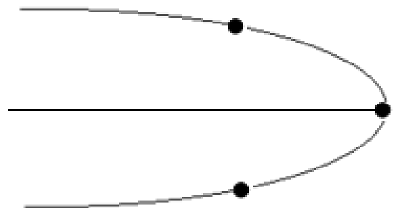

Figure 3 The singular case - example

In the singular case we can't define a function *f(x)*. Therefore we rotated about 90 degrees graph (i.e. we will consider points $[y_m, x_m]$, $[y_0, x_0]$, $[y_1, x_1]$) and calculate the tangent of function. After adjustment, it is possible to see that the equation (1) is defined the seeking function.

We can conclude that if points $[x_{i-1}, y_{i-1}]$, $[x_i, y_i]$, $[x_{i+1}, y_{i+1}]$ are not singular the function r_i is given by relation (2) and in other case by (1).

4.4. Layers of grid

The number of layers of the grid can be defined on the basis of power function:

$$y = \left(\frac{x}{n}\right)^{10}$$

where *n* is the number of layers.

Our equation has the form $r_i(k) = p_i * u(k) + q_i$, for $k=1,2,\dots,n$ and $u(k)= \left(\frac{k}{n}\right)^{10}$. Therefore, points $[x_i^k, y_i^k]$ are defined as

$$x_i^k = x_i + \left(\frac{k}{100}\right)^{10}$$

$$y_i^k = p * \left(\frac{k}{100}\right)^{10} + q$$

For k = 1, 2, …,n and i = 0, 1, ..., m.

For these points must also be restrictions $U_1 \leq x_i^k \leq D_1$ and $D_2 \leq y_i^k \leq U_2$.

Note, we can also use another function (not necessary power function) so that the density of points must be greater in the support surface and away from it was reduced.

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice

5. CONCLUSION

The next step of flow modeling is the decomposition of the problem (the division of points $[x_i^k, y_i^k]$ between processors) according to the number of processors that are available. Number of points of mesh for wing profile is *(m + 1) * n*, where *m+1* is the number of points of profile and *n* is the number of layers of mesh.

It's a simple and easy programmable system, therefore our next goal is the program implementation of the mesh algorithm in the high-performance system SIVVP in OpenFOAM.

The contribution was created on the basis of the final work [1].

6. LITERATURE LIST

References

Books:

[1] Ferencová, M.(2016) *Základy OpenFOAM*, Technická univerzita Košice, 56 p.

ISBN 978-80-553-2628-3
Faculty of Aeronautics, Technical University of Košice